Lösningsförslag

till (problem 11-17)

Tentamen i

Reglerteknik

160603

11. a

$$G_p(s) = \frac{4e^{-s}}{s} \xrightarrow[h=0{,}5\,sek]{} H_p(z) = \frac{4 \cdot 0{,}5}{z-1} z^{-2}$$

$$= \frac{2 \cdot z^{-3}}{1-z^{-1}} = \frac{B(z)}{A(z)}$$

Icke-integrerande polplacerings regulator

$grad\ P = grad\ A + grad\ B - 1 = 3$

$P(z) = (1-0{,}2z^{-1})^2(1-0z^{-1})$

$grad\ C = grad\ B - 1 = 2$ $\qquad C(z) = 1 + c_1 z^{-1} + c_2 z^{-2}$

$grad\ D = grad\ A - 1 = 0$ $\qquad D(z) = d_0$

Polynomidentiteten: $\qquad P = AC + BD$

$$1 - 0{,}4z^{-1} + 0{,}04z^{-2} = (1-z^{-1})(1+c_1z^{-1}+c_2z^{-2}) + 2z^{-3} d_0$$

z^{-0}: $1 = 1$

z^{-1}: $-0{,}4 = -1 + c_1 \quad \rightarrow \quad c_1 = 0{,}6$

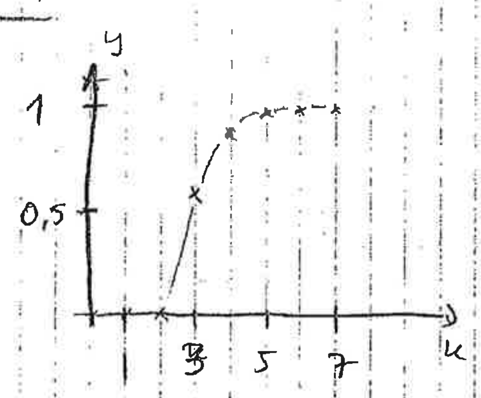
z^{-2}: $0{,}04 = -c_1 + c_2 \quad \rightarrow \quad c_2 = 0{,}04 + c_1 = 0{,}64$

z^{-3}: $0 = -c_2 + 2d_0 \quad \rightarrow \quad d_0 = 0{,}32$

$$K_r = \frac{P(1)}{B(1)} = \frac{0{,}64}{2} = 0{,}32$$

a) $\dfrac{Y}{R} = K_r \cdot \dfrac{B}{P} = \dfrac{0{,}32 \cdot 2z^{-3}}{1 - 0{,}4z^{-1} + 0{,}04z^{-2}}$ $\qquad$ Gå över till differensekvation

k	$y(k) =$	$0{,}4\,y(k-1)$	$-0{,}04\,y(k-2)$	$+0{,}64\,r(k-3)$
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0,64	0	0	0,64
4	0,896	0,256	0	0,64
5	0,9728	0,3584	−0,0256	0,64
6	0,9902	0,3891	−0,03891	0,64
7	0,9972	0,3961	−0,03891	0,64



11.b

$$V(z) = \frac{1}{C(z)}\left(K_r R(z) - D(z)\,Y(z)\right)$$

$$V(z)\left(1 + 0{,}6 z^{-1} + 0{,}64 z^{-2}\right) = 0{,}32 R(z) - 0{,}32\, Y(z)$$

Tag fram differensekvationen!

$$u(k) = -0{,}6\, u(k-1) + 0{,}64\, u(k-2) + 0{,}32\, r(k) - 0{,}32\, y(k)$$

k	u(k)				
0	0,32	0	0	+0,32	-0
1	0,128	-0,192	0	+0,32	0
2	0,0384	-0.0768	-0,2048	0,32	0
3	0,0191	-0.023	-0.0819	0,32	-0,205
4	0,0024	-0,006	-0.02458	0,32	-0,287
5	0,0008	-0,0015	-0,0064	0,32	-0,3113
6		-0,00048	+0,0015	0,32	-0,3168

11.c

Det kommer att bli ett kvarstående fel eftersom det inte finns någon integration i systemet.

$$\frac{Y(z)}{V(z)} = \frac{B/A}{1+\frac{B}{A}\cdot D\cdot\frac{1}{C}} = \frac{BC}{P} = \frac{2z^{-3}\left(1+0{,}6z^{-1}+0{,}64z^{-2}\right)}{\left(1-0{,}2z^{-1}\right)^2}$$

$$\frac{Y(1)}{V(1)} = \frac{2\cdot(1+0{,}6+0{,}64)}{0{,}8^2} = \frac{2\cdot 2{,}24}{0{,}64} = \frac{4{,}48}{0{,}64} = 7 \text{ enheter}$$

$$e_{ss} = -7 \text{ enheter}$$

12

$u[k] = 3\,e[k] + 4\,q[k]$ % PI-regulator

$e[k] = q[k] - q[k-1]$ % felsummen

a)

$K = 3$; $\frac{K}{T_i} \cdot h = 4$ $\Rightarrow T_i = \frac{3}{10}$

b) $T_d = \frac{1}{3} \cdot \frac{3}{10} = \frac{1}{10}$ sek

$$u[k] = 3\,e[k] + 4\,q[k] + \underbrace{\frac{K \cdot T_d}{h}\big(e[k] - e[k-1]\big)}_{\text{D-del.}}$$

$3 \cdot \frac{1}{10} / \frac{4}{10} = \frac{3}{4}$

13.

Räkna om förstärkningen till dB med $20\log^{10}|G|$. Antag att sensor = 1 och styrdonet också kan försummas

Ur amplitudkurvan fås en trolig överföringsfunktion enligt. $G(s) = \dfrac{K}{s(1+sT)}$

LF-asymptot $\dfrac{K}{s}$

där $K = 1$ (slår frekv.axeln vid 1 rad/s)

brytfrekvens $10\,\text{rad/s} = \dfrac{1}{T} \Rightarrow T = 0{,}1 \Rightarrow G(s) = \dfrac{1}{s(1+0{,}1s)}$

Vid titt på faskurvan inses att denna funktionen inte kan ge upphov till vår faskurva. Eftersom faskurvan skenar iväg $\to -\infty$ fasvridning.

Lägg till en dödtid. $G(s) = \dfrac{e^{-sL}}{s(1+0{,}1s)}$

Läs av fasvridningen vid någon frekvens. t.ex. $\omega = 47{,}8\,\text{rad/s}$

$\hookrightarrow \arg G(47{,}8) = -442^\circ$

$-442^\circ = -\omega L \cdot \dfrac{180^\circ}{\pi} - 90^\circ - \arctan(0{,}1\omega)$

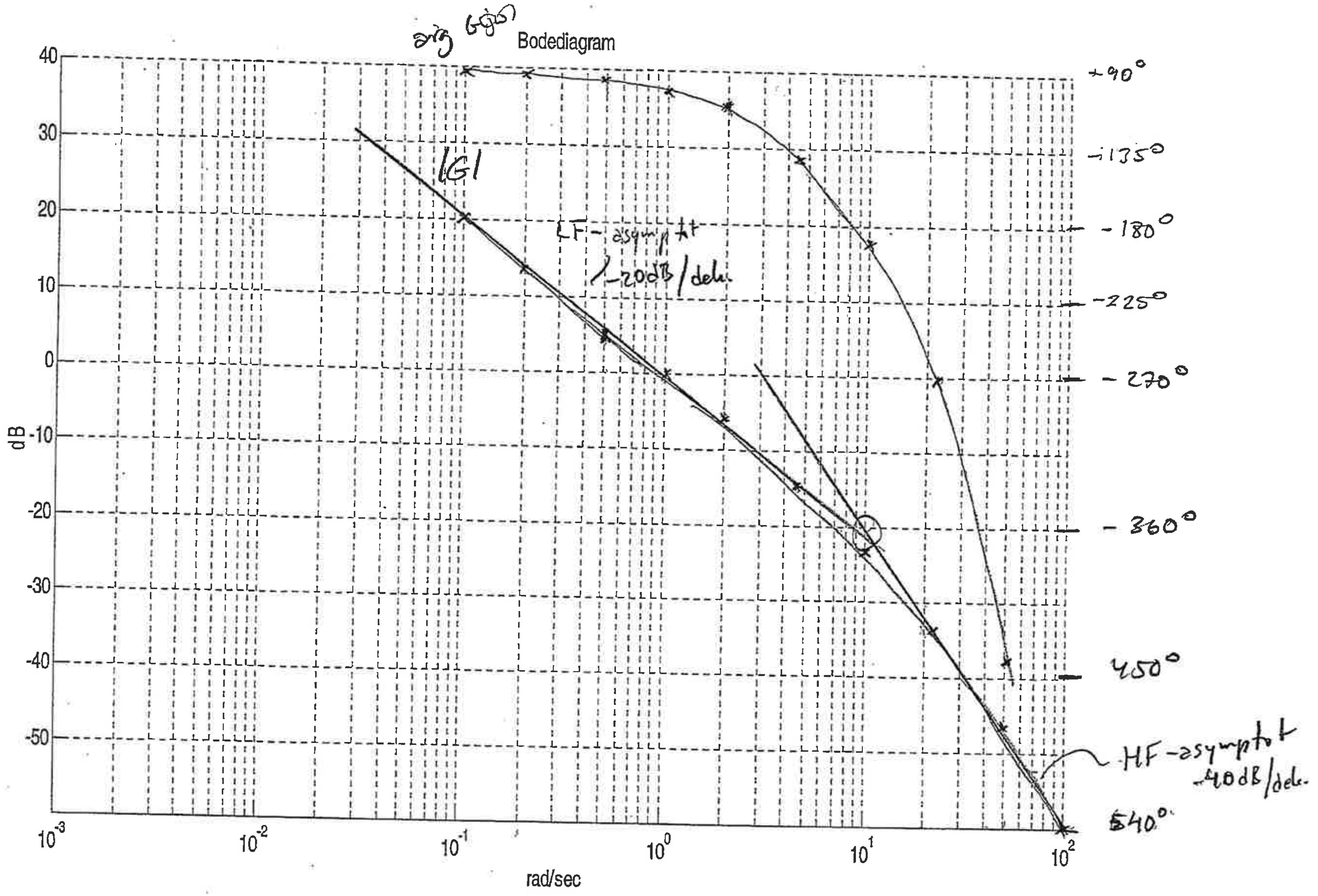
$L = -\left(-442^\circ + 90^\circ + \arctan(4{,}78)\right)\dfrac{\pi}{(180^\circ \cdot 47{,}8)} \simeq 0{,}1\,\text{sek}$

Ur Bodediagram $\omega_\pi \simeq 8\,\text{rad/s} \rightarrow A_m = 22\,\text{dB}$ $\quad \begin{cases} T_0 = \dfrac{2\pi}{\omega_\pi} \simeq 0{,}785 \\ K_0 \simeq 12{,}6 \end{cases}$

Antag P-reg = 1. $\omega_c = 1\,\text{rad/s} \rightarrow \varphi_m = 78{,}6^\circ$

Ziegler-Nichols ger: $G_{PID} \triangleq 7{,}6\left(1 + \dfrac{1}{0{,}39s} + 0{,}1s\right)$

13. forts.



arg G(jω)
Bodediagram
|G|
LF - asymptot
-20dB/dek.
HF - asymptot
-40dB/dek.
+90°
-135°
-180°
-225°
-270°
-360°
450°
540°
dB
40
30
20
10
0
-10
-20
-30
-40
-50
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
rad/sec

14.

a) Avläsning ger: $K = \frac{\Delta y}{\Delta u} = \frac{0,15}{0,4} = 0,375$

$$\begin{cases} t_p = 1,25 \text{ sek} = \frac{\pi}{\omega_0\sqrt{1-\zeta^2}} \Rightarrow \omega_0 \approx 3 \\ M = \frac{0,17-0,15}{0,15} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \Rightarrow \zeta = 0,545 \end{cases}$$

$$G(s) = \frac{K \cdot \omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{0,375 \cdot 9}{s^2 + 3,27s + 9}$$

b) Avläsning ger $t_{1/3} = 1,85$ sek, $t_{2/3} = 3,7$ sek, $Q = \frac{t_{2/3}}{t_{1/3}} = 2,06$

ur figur 7.1: $a = 0,32$, $p = 1,11$ $\Rightarrow$ $T = \frac{t_{2/3}}{p(1+a)} \approx 2,53$

$$G(s) = \frac{K}{(1+sT)(1+aTs)} = \frac{3,5/0,7}{(1+2,53s)(1+0,81s)}$$

c) Avläsning ur A) $t_p = 1,25$ sek

$t_{s5\%} = 2$ sek

$t_r = 0,75 - 0,15 = 0,6$ sek

$M \approx \frac{0,02}{0,15} = 13,3\%$

B) t_p saknas, M saknas

$t_r = 7,5 - 0,5 \approx 7$ sek

$t_{s5\%} \approx 10$ sek

15.

a) $G_r = \dfrac{Y(s)}{D(s)} = \dfrac{-\dfrac{40}{s(s+20)}}{1 + K\cdot\dfrac{100}{(s+10)}\cdot\dfrac{40}{s(s+20)}} =$

$$= -\frac{40(s+10)}{(s+10)\,s(s+20) + 4000K}$$

$|Gr(0)| \leq 0{,}05 \iff \dfrac{400}{4000K} < 0{,}05$

$$\boxed{2 < K}$$

b) Karakteristisk ekvation $1 + K\cdot\dfrac{100}{(s+10)}\cdot\dfrac{40}{s(s+20)} = 0$

$(s+10)(s^2+20s) + 4000K = 0$

$s^3 + 20s^2 + 10s^2 + 200s + 4000K = 0$

$s^3 + 30s^2 + 200s + 4000K = 0$

Routh-Hurwitz:

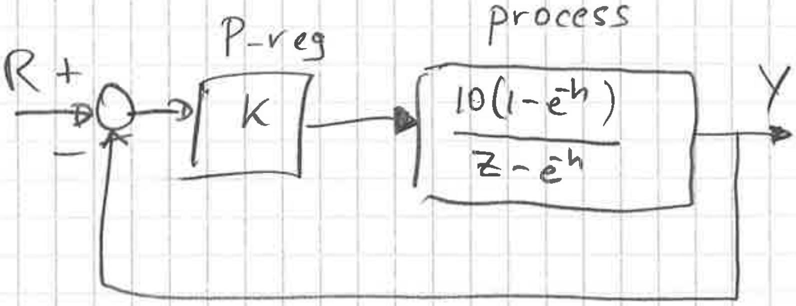
s^3	1	200	0
s^2	30	$4000K$	0
s^1	$\dfrac{4000K - 6000}{30}$	0	$\rightarrow K > \dfrac{3}{2}$
s^0	$4000K$		$\rightarrow K > 0$

16.

$$G_p(s) = \frac{10}{s+1} \xrightarrow{h=?} H_p(z) = \frac{10(1-e^{-h/1})}{z-e^{-h/1}}$$

$H_R = K$

Vårt reglersystem:



Karakteristisk ekvation:

$$1 + K \cdot 10 \frac{(1-e^{-h})}{z-e^{-h}} = 0$$

$$z - e^{-h} + 10K(1-e^{-h}) = 0$$

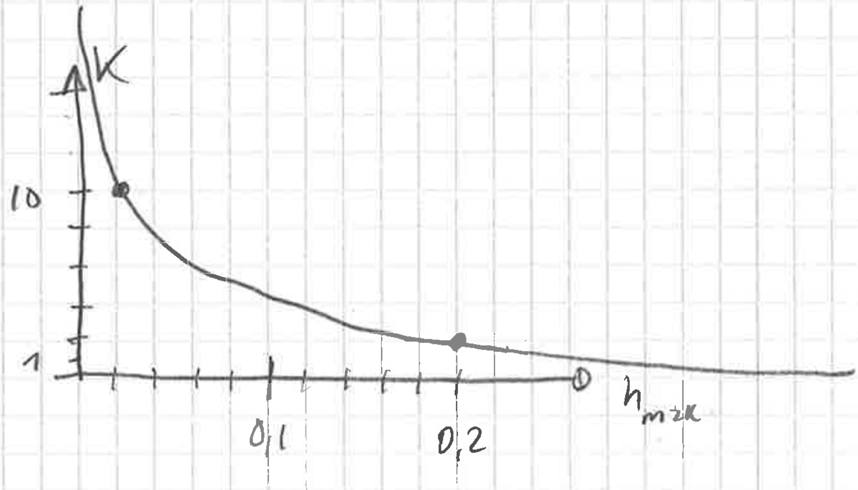
$$z = (1+10K)e^{-h} - 10K$$

$K=0{,}1$: $z = 2e^{-h} - 1 \quad z=-1$ h_{max} saknas.

$K=1$: $z = 11e^{-h} - 10 = -1 \quad h_{max} = -\ln\frac{9}{11} \approx 0{,}2$ sek

$K=10$: $\begin{cases} z = 101e^{-h} - 100 \\ z=-1 = 101e^{-h} - 100 \end{cases}$

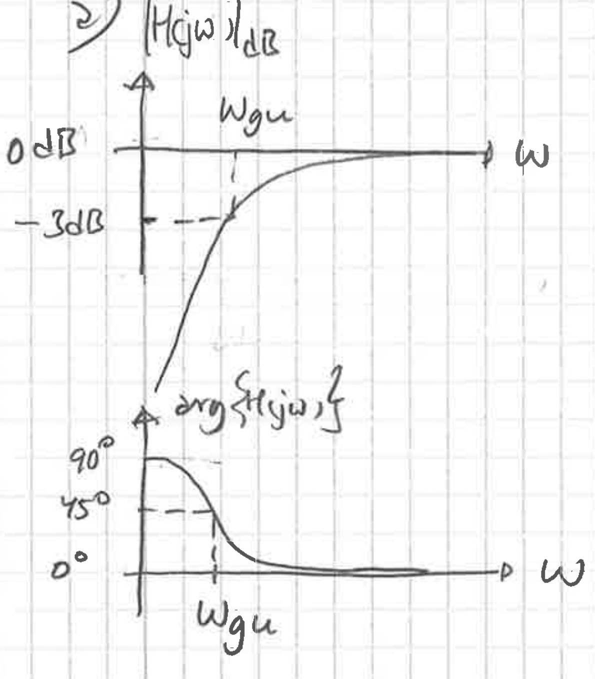
$$h_{max} = -\ln\frac{99}{101} = 0.02 \text{ sek}$$



17.

$$H(j\omega) = \frac{U_{ut}}{U_{in}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{j\omega RC + 1}$$

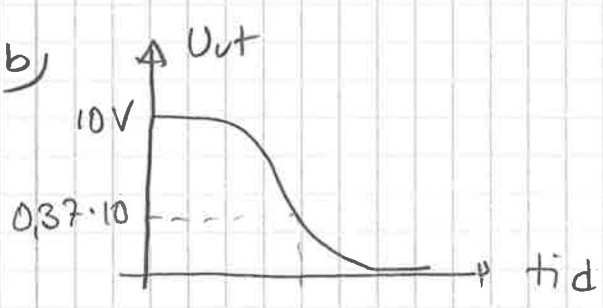
a)



$$\omega_{gu} = \frac{1}{RC} =$$

$$f_{gu} = \frac{1}{2\pi RC}$$

b)



$$U_{ut} = 10e^{-t/\tau}$$

der $\tau = RC = 1\,msek$